

Engineering of multi-dimensional entangled states of photon pairs using hyper-entanglement

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Multi-dimensional entangled states have been proven to be more powerful in some quantum information process. In this paper, down-converted photons from spontaneous parametric down conversion (SPDC) are used to engineer multi-dimensional entangled states. A kind of multi-degree multi-dimensional Greenberger-Horne-Zeilinger (GHZ) state can also be generated. The hyper-entangled photons, which are entangled in energy-time, polarization and orbital angular momentum (OAM), is proved to be useful to increase the dimension of systems and investigate higher-dimensional entangled states.

PACS numbers: 03.67.Mn, 03.65.Ud, 42.50.Dv

I. INTRODUCTION

Quantum entanglement is the foundation of quantum teleportation, quantum computation, quantum cryptography, superdense coding, etc. In recent years, the interest in multi-dimensional entangled states, or qudits, is steadily growing. One advantage of using multilevel systems is its promise to realize new types of quantum communication protocols[1, 2, 3], and its better properties in quantum cryptography than qubits[2, 3, 4, 5]. These cryptography protocols are more robust against specific classes of eavesdropping attacks. The other advantage is their possible implementation in the fundamental tests of quantum mechanics. For two-partite system of dimension greater than two, it has been found that the Clauser-Horne-Shimony-Holt (CHSH) inequality can be maximally violated, and this violation continues to survive in the limit of infinite dimensions[6]. GHZ paradoxes and Bell inequality have also been discussed for multi-partite systems of multi-dimension in[7, 8, 9, 10, 11]. Additionally, the usage of multilevel systems provides a possibility to introduce very special protocol, which cannot be implemented with qubits, such as quantum bit commitment[12].

There are several approaches to investigate higher dimensional systems. In one case, multiple entangled qubit systems are used, such as four-photon polarization scheme[13]. In some other cases, dimensions of the element are increased. For example, the interferometer method was used to generate energy-time entangled qutrits[14, 15]; and other techniques rely on the spatial modes of down-converted photons from SPDC. These down-converted photons can be entangled in not only polarization, or spin angular momentum, but also spatial modes, such as orbital angular momentum (OAM)[16], Hermite-Gaussian modes[17]. The spatial entanglement

occurs in an infinite-dimensional Hilbert space. Many theoretical[18, 19, 20, 21] and experimental[16, 23, 24] works about entangled qudits have been done based on OAM of the photons.

Quantum state engineering, i.e., the ability to generate, transmit and measure quantum systems is of great importance in quantum information process. Qutrit state engineering has been done using polarization of biphotons[25]. In the present protocol, we show that any entangled qudit states can be generated by exploiting the OAM and polarization of the energy-time entangled photons. A kind of artificial multi-partite multi-dimensional entangled GHZ states is also generated by the help of photon pairs entangled in polarization, OAM and energy-time simultaneously. (This kind of multiply-entanglement is named as hyper-entanglement[26].) Although only two photons are involved in these GHZ states, they appear like multi-partite entangled states due to the usage of multi-degree of each photon. We call this kind of states

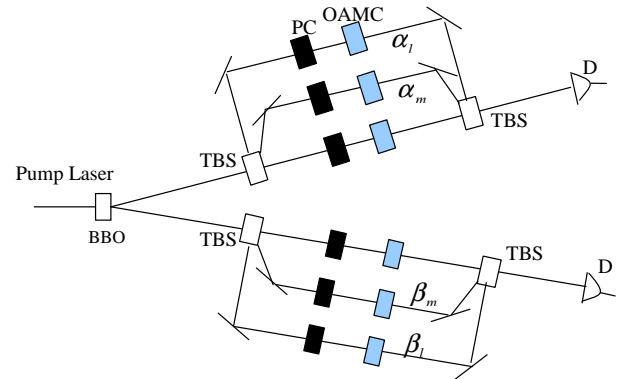


FIG. 1: Schematic experimental setup for generation of multi-freedom multi-dimensional entangled states. In each path, there is a three-arm interferometers. TBS is three-beam-splitter which can also be substituted by two BS. PCs and OAMCs are added in each arm to control the polarization and OAM of down-converted photons.

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multi-degree multi-dimensional entangled states.

II. EXPERIMENTAL SETUP TO GENERATE ENTANGLED QUDITS AND MULTI-DEGREE MULTI-DIMENSIONAL GHZ STATES

Energy-time entangled qutrit states can be generated as photon pairs from SPDC passing two three-arm interferometers[14, 15]. For each interferometer, a phase vector consisting of two independent phases can be defined, e.g., the relative phase between the short(s)-medium(m) and short-long(l), path lengths. Coincidence measurement at the outputs of the interferometers project onto entangled qutrit states defined when the photons take the same arm in each interferometer, short-short, medium-medium or long-long at signal-idler path. For this type of experiments, the following conditions must be satisfied[15]: The coherence length of the down-converted photons is much smaller than the path-length difference in the interferometers so that no single photon interference effects are observed in passing the interferometers; the coherence length of the pump laser is much greater than these path-length differences so that we have no timing information as to the creation time of the photon pairs and hence which path was taken before detection.

As polarization control(PC) and OAM control (OAMC) are added, we extend the experimental set-up of Thew and his co-workers[14, 15] to Fig. 1. Consider the case that the photons take the same path in each interferometer, the state can be written as:

$$|\psi\rangle \propto c_s |ss\rangle + c_m e^{i(\alpha_m + \beta_m + \Phi_{jk}^m)} |mm\rangle + c_l e^{i(\alpha_l + \beta_l + \Phi_{jk}^l)} |ll\rangle, \quad (1)$$

where α_m, α_l and β_m, β_l represent the phase in medium and long interferometer arms of signal and idler path, Φ_{jk}^m and Φ_{jk}^l are multiples of $2\pi/3$ which depend on the path taken by the photons in the interferometer and the output, $j, k \in \{0, 1, 2\}_{A,B}$, they taken. Apparently, we can justify the state of Eq. (1) to:

$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{3}}(|ss\rangle + |mm\rangle + |ll\rangle) \\ &= \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle). \end{aligned} \quad (2)$$

It is a maximally entangled GHZ state of two qutrits[14, 15].

A. Generation of qudits and three-degree three-dimensional GHZ state

The Polarization control(PC)(see Fig. 2a) is a device that can be used to control the polarization of pho-

TABLE I: The polarization of photons in each arm of signal and idler path.

Signal path	Polarization	Idler path	Polarization
Short arm	H	Short arm	H
Medium arm	H	Medium arm	V
Long arm	V	Long arm	V

tons. This can be realized by the combination of Polarization Beam Splitter(PBS), Half Wave Plate(HWP) and Quarter Wave Plate(QWP). The polarization of photons passing through PBS can be determinately H or V. Using HWP and QWP, we can get any polarization we wanted. If the polarization of the down-converted photons is known, for example, the crystal is cut for Type-1 SPDC, there is no need to use PBS.

With PCs, we can change the polarization of the photons according to the paths they taken. For instance, photons with polarization showing in Tab. 1 can be achieved. In this case, the state of Eq. (2) will change to:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0_H 0_H\rangle + |1_H 1_V\rangle + |2_V 2_V\rangle). \quad (3)$$

A set of basis states can be defined with the polarization of the two photons[25]: $|HH\rangle = |0\rangle$; $|HV\rangle = |1\rangle$; $|VV\rangle = |2\rangle$. If just consider the information of polarization, we get the state:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle). \quad (4)$$

Obviously, we can control the polarization of photons in each path to change this state to arbitrary qutrit: $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$. If we additionally define $|VH\rangle = |3\rangle$, we will get arbitrary 4-dimensional state: $\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \delta|3\rangle$.

Thus we can get the three-degree three-dimensional entangled GHZ state:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle), \quad (5)$$

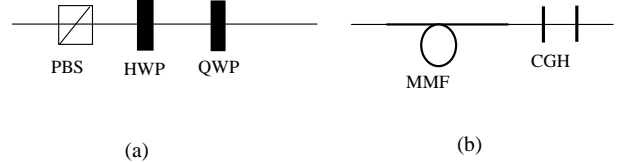


FIG. 2: (a) is a PC which is used to control the polarization of photons. It includes PBS, HWP and QWP. (b) is an OAMC which is used to control the OAM of Photons. It includes MMF and CGHs. The PBS (MMF) can be taken out if we know the polarization (OAM) of the input photon.

where the first two terms of each $|nnn\rangle$ ($n = 0, 1, 2$) represent the path information and the third term represents the polarization information.

B. Generation of entangled qutrits and four-degree three-dimensional GHZ state

Now we will discuss the function of OAM control(OAMC). The OAMC(see Fig. 2b) is a device that can be used to control the OAM of photons. It has been shown that paraxial Laguerre-Gaussian(LG) laser beams carry a well-defined orbital angular momentum[22], and the LG modes form a complete Hilbert space. Every photon of the pure LG_p^l mode beam carries an OAM of $l\hbar$. Many experimental works[16, 23, 24] about qutrits are based on OAM of the photons since the down converted photons from SPDC are entangled in OAM. In these works, computer generated hologram(CGH) and mono-mode fiber(MMF) were used to generate and detect high order LG modes. Only the photons with 0 OAM can pass the MMF, all other modes have a large spatial extension, and therefore cannot be coupled into MMF. The function of MMF to OAM is like PBS to polarization. The CGHs are used to change the OAM of photons. By shifting the CGHs[27] or designing special CGHs[28], we can generate superposition of different OAM states. The OAMC can be realized by the combination of MMF and CGHs. The efficiency of this kind OAMC is lower than 1.

By the help of OAMCs, and just consider the information of OAM of the photons, we can engineer any pure two-partite three-dimensional entangled states:

$$|\psi\rangle = |0\rangle(\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle) + |1\rangle(\alpha_{10}|0\rangle + \alpha_{11}|1\rangle + \alpha_{12}|2\rangle) + |2\rangle(\alpha_{20}|0\rangle + \alpha_{21}|1\rangle + \alpha_{22}|2\rangle), \quad (6)$$

where $|0\rangle, |1\rangle$ and $|2\rangle$ is the OAM of the photons.

In detail, pure two-partite three-dimensional entangled states can be divided into three group. First, it only includes the terms in one of the three lines of Eq. (6), for example $|0\rangle(\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle)$. We can get this type of states by controlling the photons taken the signal path in the state $|0\rangle$, and photons taken the idler path in the state $\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle$. Second, the state includes the terms coming from two lines of Eq.(6), such as $|0\rangle(\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle) + |1\rangle(\alpha_{10}|0\rangle + \alpha_{11}|1\rangle + \alpha_{12}|2\rangle)$. This state can be generated when the photons taken the short arm and medium arm of signal path have 0 OAM, photons taken the long arm of signal path have 1 OAM, and photons taken the short arm and medium arm of idler path are in the state $1/2(\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle)$, photons taken the long arm of idler path is in the state $\alpha_{10}|0\rangle + \alpha_{11}|1\rangle + \alpha_{12}|2\rangle$. Third, the state includes terms coming from all three lines of Eq. (6), for example $|0\rangle(\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle) + |1\rangle(\alpha_{10}|0\rangle + \alpha_{11}|1\rangle +$

$\alpha_{12}|2\rangle) + |2\rangle(\alpha_{20}|0\rangle + \alpha_{21}|1\rangle + \alpha_{22}|2\rangle)$. In this case, the photons taken the signal short, medium and long arm of signal path can be controlled to have 0, 1, 2 OAM, and the photons taken the short, medium and long arm of idler path in state $\alpha_{00}|0\rangle + \alpha_{01}|1\rangle + \alpha_{02}|2\rangle, \alpha_{10}|0\rangle + \alpha_{11}|1\rangle + \alpha_{12}|2\rangle$ and $\alpha_{20}|0\rangle + \alpha_{21}|1\rangle + \alpha_{22}|2\rangle$ respectively. So we can get arbitrary pure two-partite three-dimensional entangled state.

Consider the information of both the paths and OAM of photons, we can obviously get the four-degree three-dimensional GHZ state:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|0000\rangle + |1111\rangle + |2222\rangle), \quad (7)$$

where the first two terms of $|nnnn\rangle$ ($n = 0, 1, 2$) carry the information of the paths and last two terms carry the information of OAM of photons.

C. Generation of multi-dimensional entangled states

It has been proven that angular momentum can be separate into orbital and spin contributions[29]. So we can use OAMCs and PCs at the same time. In this situation, we can engineer arbitrary pure three-degree three-dimensional entangled states:

$$|\psi\rangle = \sum_{m,n,p=0}^2 \alpha_{mnp} |mnp\rangle. \quad (8)$$

Similarly, we can easily get the state:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00000\rangle + |11111\rangle + |22222\rangle). \quad (9)$$

It is a five-degree three-dimensional maximally entangled GHZ state.

In fact, we will find these multi-degree multi-dimensional entangled states can also be two-partite multi-dimensional entangled states, while the dimension is $m \times n \times p \times \dots$, where m is the dimension of the first freedom, n the second, p the third and so on. For example, we can do the following definition: $|0\rangle = |0_H\rangle, |1\rangle = |0_V\rangle, |2\rangle = |1_H\rangle, |3\rangle = |1_V\rangle, |4\rangle = |2_H\rangle, |5\rangle = |2_V\rangle$, where $|0_H\rangle$ represent the OAM of the photon is 0 and polarization H , others similarly. This kind of states can be measured by combination of PBS, HWP, QWP, CGH, and MMF. Then the state of Eq. (8) will change to a two-partite six-dimensional(3×2) entangled state:

$$|\psi\rangle = \sum_{m,n=0}^5 \alpha_{mn} |mn\rangle. \quad (10)$$

Obviously, if we can use more freedom of photons, we can easily increase the dimension.

Bell inequalities and CHSH inequalities have been developed to arbitrarily high-dimensional systems[6, 30]. To test these inequalities, we need to think the architecture to increase the dimension of the systems. For example, in the OAM schemes, lens must be used to get the maximally entangled states. It will be difficult to realize if higher dimension is needed. The present protocol shows that it seems much more convenient to increase the dimension of systems using hyper-entanglement.

III. CONCLUSION

In conclusion, different freedoms(path, polarization and OAM) of down-converted photons from SPDC are used to generate multi-dimensional entangled states. The

photon pairs are entangled in energy-time, polarization and OAM which is called hyper entangled. We can engineer multi-dimensional states and multi-dimensional entangled states of two partite systems in this way. Multi-degree multi-dimensional GHZ states can also be generated using this hyper entanglement. It is easier to control this entangled qudits than we just use high dimension of single freedom of photons. The hyper-entanglement is proved to be useful for investigation of higher-dimensional entangled states.

Acknowledgments

This work was funded by the Chinese National Fundamental Research Program (2001CB309300), the National Natural Science Foundation(Grant No.60121503), the NSF of China(10304017), the Innovation Funds from Chinese Academy of Sciences.

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